The MRMI Algorithm

The batch mode adaptation algorithm for the rotation matrix, which is parameterized in terms of Givens rotations, can be summarized as follows.

1. Whiten the observations $\{z_1, \ldots, z_N\}$ using **W** to produce the samples $\{x_1, \ldots, x_N\}$.
2. Initialize (randomly) the Givens rotation angles θ_{ij}, $i = 1, \ldots, n-1$, $j = i+1, \ldots, n$.
3. Compute the rotation matrix using Eq. (8.56) and evaluate the output samples.
4. Until the algorithm converges repeat the following steepest descent procedure.
   a. Evaluate the gradient of the cost function $J(\theta) = \sum_{o=1}^{M} \hat{H}_\alpha(Y_o)$, using

$$\frac{\partial J}{\partial \theta_{ij}} = \sum_{o=1}^{M} \frac{\partial \hat{H}_\alpha(Y_o)}{\partial \theta_{ij}} = \sum_{o=1}^{M} \frac{1}{1-\alpha} \frac{\partial \hat{V}_\alpha(Y_o)/\partial \theta_{ij}}{\hat{V}_\alpha(Y_o)}, \tag{8.57}$$

   where the information force $\partial \hat{V}_\alpha / \partial y_o$ is estimated by Eq. (2.69) and

$$\mathbf{y}_{o,j} = \mathbf{R}^o \mathbf{x}_j \;, \quad o = 1, \ldots, M, \quad j = 1, \ldots, N$$

$$\frac{\partial y_{o,j}}{\partial \theta_{ij}} = \frac{\partial \mathbf{R}^o}{\partial \theta_{ij}} \mathbf{x}_j = \left(\frac{\partial \mathbf{R}}{\partial \theta_{ij}} \right)^o \mathbf{x}_j \tag{8.58}$$

$$\frac{\partial \mathbf{R}}{\partial \theta_{ij}} = \left(\prod_{p=1}^{i-1} \prod_{q=p+1}^{M} \mathbf{R}_{pq} \right) \left(\prod_{q=i}^{j-1} \mathbf{R}_{iq} \right) \mathbf{R}'_{ij} \left(\prod_{q=j+1}^{M} \mathbf{R}_{iq} \right) \left(\prod_{p=i+1}^{M-1} \prod_{q=p+1}^{M} \mathbf{R}_{pq} \right), \tag{8.59}$$

   where for any matrix **A**, $\mathbf{A}^o$ denotes the *o*th row of that matrix and $\mathbf{R}'_{ij}$ denotes the derivative of the specific Givens rotation matrix (in the i, j plane) with respect to its parameter θ_{ij}.

   b. Evaluate the sign of the sum of kurtosis (K), and update the Givens angles using

$$\theta_{ij} \leftarrow \theta_{ij} - \eta \ \text{sign}(K) \frac{\partial J}{\partial \theta_{ij}}. \tag{8.60}$$